Multiple-antenna Placement Delivery Array with Cyclic Placement

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***Abstract*—This paper studies the cache-aided multiple-input single-output (MISO) broadcast problem with one-shot linear delivery, where a server with L antennas and N files is connected to K single-antenna users (each with a memory of M files) through a wireless broadcast channel, with the objective to maximize the sum Degree-of-Freedom (sum-DoF) in the whole system. Recently, a construction structure, referred to as Multiple-antenna Placement Delivery Array (MAPDA), was proposed to construct coded caching schemes for this cache-aided MISO broadcast problem based on the joint design of coded caching and zero-forcing (ZF) precoding. In this paper, we first propose an upper bound on the sum-DoF of any MAPDA scheme given a fixed cache placement. Then, under a class of cyclic placements which leads to subpacketizations on the files linear with K, we propose two MAPDAs for the case $L < KM/N$ achieving the sum-DoF $2L$, which is order optimal within a factor of 2 when $M/N \leq 1/2$ compared to the upper bound under the cyclic placement.**

I. Introduction and Problem Description

Coded caching originally proposed by Maddah-Ali and Niesen (MN) in [1], enables the transmission of multicast messages which can serve multiple users simultaneously by leveraging the users' cached content to remove interference, such that the communication load/time could be significantly reduced. In this paper, we consider the K-user cache-aided MISO broadcast problem with one-shot linear delivery originally studied in [2], where an L-antenna transmitter communicates to K cache-aided single-antenna users through a wireless broadcast channel. The server is equipped with $N \geq K$ files $W_1, \ldots, W_N$, where each file W_n has F packets and each packet, denoted by $W_{n,f}$, contains B uniform and i.i.d. bits where $n \in \{1, \ldots, N\}$ and $f \in \{1, \ldots, F\}$. Each user $k \in \{1, \ldots, K\}$ has a cache $\mathcal{Z}_k$ to store up to M files. The broadcast channel is modelled as follows. At each time slot of transmission, say t-th time slot, the server sends one signal $X_i(t) \in \mathbb{C}$ through the i^{th} antenna for each $i \in \{1, \ldots, L\}$, under the power constraint $\mathbb{E}\big[\sum_{1 \leq i \leq L} |X_i(t)|^2\big] \leq P$, where P is assumed to be large enough (i.e., high SNR regime). The received signal at this time slot by user k is $Y_k(t) = \sum_{i=1}^{L} h_{k,i} X_i(t) + \epsilon_k(t)$, where $h_{k,i} \in \mathbb{C}$ denotes the channel gain between antenna i and user k, which is assumed to remain unchanged in the whole communication process and perfectly known to the server and all users, and $\epsilon_k(t) \sim \mathcal{CN}(0,1)$ represents the noise.

A coded caching scenario has two phases. During the placement phase, each user fills its cache without knowledge of future demand. During the delivery phase, each user $k \in \{1, \ldots, K\}$ requests one file W_{d_k}, where $d_k \in \{1, \ldots, N\}$. The server encodes each packet $W_{n,f}$ into a coded packet $\tilde{W}_{n,f}$ by using a code for the Gaussian channel with rate $B/\tilde{B} = \log P + o(\log P)$ (bit per complex symbol). Then the transmission is divided into S blocks, each of which contains $\tilde{B}$ time slots. In each block $s \in \{1, \ldots, S\}$, the one-shot transmission from the server is useful to r_s users, each of which can recover one desired coded packet. Since each coded packet carries one DoF, the sum-DoF of the whole system is $\sum_{s=1}^{S} r_s/S$. The objective of this problem is to maximize the worst-case sum-DoF among all possible demands.

For this cache-aided MISO broadcast problem with one-shot linear delivery, the authors in [2]–[9] proposed achievable caching schemes by combining coded caching and ZF. Under the constraint of uncoded cache placement and one-shot linear delivery, the maximum sum-DoF is $L + \frac{KM}{N}$ [10]. For the general case, the sum-DoF $L + \frac{KM}{N}$ is achieved in [2], [4], while suffering from extremely high subpacketzations (which are even more than $\binom{K}{KM/N}\binom{K-KM/N-1}{L-1}$). Various achievable caching schemes were proposed in [5]–[9] under different system parameters constraints, which can reduce the subpacketzation of the above schemes. For example, a coded caching scheme which can achieve the sum-DoF $L + \frac{KM}{N}$ with a linear subpacketization in terms of K, was proposed in [6], under the constraint that $L \geq KM/N$.

Placement delivery array (PDA) originally proposed in [11], is a combinatorial structure to construct coded caching schemes with limited subpacketization for the shared-link systems. Very recently, the authors in [12], [13] extended the concept of PDA to the cache-aided MISO broadcast problem with one-shot linear delivery, and introduced a novel combinatorial based on uncoded cache placement, referred to as *Multiple-antenna Placement Delivery Array (MAPDA)*, which can represent the schemes in [2]–[9]. Based on the MAPDA structure, some achievable schemes which can further reduce the subpakcetization while achieving the optimal sum-DoF $L + \frac{KM}{N}$, were proposed in [12], [13]. More precisely, in [12] an MAPDA design was proposed for the general case which reduces the subpacketizations of those in [2], [4]. Another MAPDA design was propsoed in [13] for the case $L \geq KM/N$, with lower linear subpacketization than [6].

Our Contributions: In this paper, we first propose an upper bound on the sum-DoF of any MAPDA given a fixed placement strategy. We then introduce a placement class, referred to as cyclic placement, where the packets of each file is cached by the users cyclically. Based on the cyclic placement, we propose two MAPDAs for the case where $L < KM/N$, whose achieved sum-DoF is equal to $2L$ and the needed subpacketizations are linear with K. Note that the existing caching schemes with linear subpacketization for this cache-aided MISO broadcast problem need the constraint that $L \geq KM/N$. Finally, we prove that the achieved sum-DoF by the proposed MAPDA is order optimal within a factor of 2 when $M/N \leq 1/2$, under the constraint of the cyclic placement.

Notations: We let $[a:b] = \{a, a+1, \cdots, b\}$ and $[a] = \{1, 2, \cdots, a\}$; the sum of a scalar and a vector represents the vector obtained by incrementing each element of the vector by the scalar; $< \cdot >_a$ represents the modulo operation with integer quotient $a > 0$ and in this paper we let $< \cdot >_a \in \{1, \ldots, a\}$ (i.e., we let $< b >_a = a$ if a divides b); if $\cdot$ is a vector, $< \cdot >_a$ represents the vector obtained by taking the modulo operation with integer quotient a on each element of this vector.

II. Multiple-Antenna Placement Delivery Array

We review the definition of MAPDA proposed in [12]. Note that we consider the case where $L \geq 2$, since the case where $L = 1$ reduces to the MN shared-link coded caching problem.

Definition 1 ([12]). *For any positive integers L, K, F, Z and S, an $F \times K$ array $\mathbf{P} = (p_{f,k})_{f\in[F],k\in[K]}$ composed of "$*$" and $[S]$, is called (L, K, F, Z, S) MAPDA, if it satisfies the following conditions*

C1. *The symbol "$*$" appears Z times in each column;*
C2. *Each integer $s \in [S]$ appears at least once in the array;*
C3. *Each integer $s \in [S]$ appears at most once in each column;*
C4. *For any integer $s \in [S]$, define $\mathbf{P}^{(s)}$ as the subarray of $\mathbf{P}$ composed of the rows and columns containing s, and $r'_s \times r_s$ as the dimension of $\mathbf{P}^{(s)} = (p^{(s)}_{f,k})_{f\in[r'_s],k\in[r_s]}$. The number of non-star entries in each row of $\mathbf{P}^{(s)}$ is less than or equal to L, i.e.,*

$$\left|\{k \in [r_s] | \ p^{(s)}_{f,k} \in [S]\}\right| \leq L, \ \forall f \in [r'_s]. \quad (1)$$

If each integer appears g times in the $\mathbf{P}$, then $\mathbf{P}$ is regular, denoted by g-(L, K, F, Z, S) MAPDA. For instance, the following array is a 4-$(3, 4, 4, 1, 3)$ MAPDA,

$$\mathbf{P} = \begin{pmatrix} * & 1 & 2 & 3 \\ 1 & * & 3 & 2 \\ 2 & 3 & * & 1 \\ 3 & 2 & 1 & * \end{pmatrix}.$$

It was shown in [12, Theorem 1] that given an MAPDA $\mathbf{P}$, we can obtain a coded caching scheme for the cache-aided MISO broadcast problem.

Theorem 1 ([12]). *For a given (L, K, F, Z, S) MAPDA, there exists an F-division scheme for the (L, K, M, N) cache-aided MISO broadcast problem with memory ratio $\frac{M}{N} = \frac{Z}{F}$, sum-DoF $\frac{K(F-Z)}{S}$, and subpacketization F.*

It was also shown in [10, Corollary 1] that, the maximum sum-DoF achieved by the schemes under MAPDA is $L + \frac{KM}{N}$.

Note that, for any (L, K, F, Z, S) MAPDA satisfying the conditions in Definition 1, if we replace the non-star entries in the array by null entries, the resulting array is called a (K, F, Z) star placement array, which represents the placement phase of the coded caching scheme obtained from the MAPDA.

III. Main Results and Numerical Evaluations

In this section, for a given star placement array, we first derive a lower bound on the number of integers in any MAPDA under this star placement array. Then two MAPDAs under a cyclic star placement, which is defined in the following Definition 2, are constructed for the case where $L < KM/N$. Compared to the proposed lower bound, the sum-DoF achieved by our proposed MAPDAs is at least half of that of the optimal MAPDA under the cyclic star placement when $M/N \leq 1/2$.

Definition 2. *(Cyclic star placement) A (K, F, Z) star placement array $\mathbf{P}' = (p'_{f,k})_{f\in[F],k\in[K]}$ including stars and null entries, is referred to as a cyclic star placement array, if F is divisible by K, $t := \frac{KZ}{F}$ is an integer, and the stars in each row are placed in a cyclic wrap-around topology, i.e., each entry*

$$p'_{f,k} = *, \quad \text{only if} \ \ k \in \{< f + z >_K \ | \ z \in [0 : t-1]\}. \quad (2)$$

For instance, we can check that the following array

$$\mathbf{P}' = \begin{pmatrix} * & * & & \\ & * & * & \\ & & * & * \\ * & & & * \end{pmatrix}$$

is a $(4, 4, 2)$ star placement array.

A. Upper bounds on the sum-DoF

Given an MAPDA or a star placement array $\mathbf{P} = (p_{f,k})_{f\in[F],k\in[K]}$, we denote the set of the indices of rows, each of which contains a non-star entry in the k-th column, by

$$\mathcal{A}_k = \{f \in [F] \mid p_{f,k} \neq *\}, \ \forall k \in [K]. \quad (3)$$

From the above notation, the following statement holds.

Theorem 2. *Let $\mathcal{I}$ be the permutation set of $[K]$. Given an (L, K, F, Z) star placement array, the sum-DoF of any (L, K, F, Z, S) MAPDA under this star placement array should be no larger than*

$$\frac{K(F-Z)}{\left\lceil \frac{1}{L} \cdot \max\left\{ \sum_{i=1}^{K} \left| \bigcap_{h=1}^{i} \mathcal{A}_{k_h} \right| \ \middle| \ (k_1, k_2, \ldots, k_K) \in \mathcal{I} \right\} \right\rceil}.$$

Proof. Focus on any (L, K, F, Z, S) MAPDA under the given star placement array with $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_K$ defined in (3). For any g distinct column indices, $k_1, k_2, \ldots, k_g$, the following statements can be obtained according to the values of g and L.

- When $g \leq L$, from Condition C3 of Definition 1, all the non-star entries in the column labeled by k_1 are different. Then we have $S \geq |A_{k_1}| \geq \frac{1}{L}\left(|\mathcal{A}_{k_1}| + |\mathcal{A}_{k_1} \bigcap \mathcal{A}_{k_2}| + \cdots + |\bigcap_{i=1}^{g} \mathcal{A}_{k_i}|\right)$.
- When $g > L$, for any $L+1$ row indices, $f_1 \in \mathcal{A}_{k_1}$, $f_2 \in \mathcal{A}_{k_1} \cap \mathcal{A}_{k_2}$, …, $f_{L+1} \in \bigcap_{i=1}^{L+1} \mathcal{A}_{k_i}$, by the definition in (3), each of p_{f_1,k_1}, p_{f_2,k_2}, …, $p_{f_{L+1},k_{L+1}}$ is an integer. Furthermore, by Condition C4 of Definition 1 (i.e., for any integer $s \in [S]$, the number of non-star entries in each row of $\mathbf{P}^{(s)}$ which is composed of the rows and columns containing s, is less than or equal to L), $p_{f_1,k_1}, p_{f_2,k_2}, \ldots, p_{f_{L+1},k_{L+1}}$ cannot be the same.[1] In other words, in the positions (f_i, k_i) of the MAPDA where $f_i \in \bigcap_{h=1}^{i} \mathcal{A}_{k_h}$ and $i \in [g]$, each integer appears at most L times. As a result, the number of distinct integers in the MAPDA is $S \geq \lceil \frac{1}{L}\left(|\mathcal{A}_{k_1}| + |\mathcal{A}_{k_1} \bigcap \mathcal{A}_{k_2}| + \cdots + |\bigcap_{i=1}^{K} \mathcal{A}_{k_i}|\right) \rceil$.

Since the sum-DoF is $K(F-Z)/S$, the proof is complete. □

For any cyclic star placement, the following result can be directly obtained from Theorem 2.

Corollary 1. *Given a (K, F, Z) cyclic star placement array where F is divisible by K and $t := KZ/F$, the sum-DoF of any (L, K, F, Z, S) MAPDA under this star placement array should be no larger than $\frac{K(F-Z)}{\lceil \frac{F(K-t)(K-t+1)}{2LK} \rceil}$.*

Proof. For any (L, K, F, Z, S) MAPDA under a given (L, K, F, Z) cyclic star placement array, from (2) for each integer $k \in [K]$, we have the set of row indices which contains a non-star entry in the k-th column $\mathcal{A}_k = \{< k + z >_K + gK \,\Big|\, z \in \left[K - \frac{KZ}{F} + 1, K\right], g \in \left[0, \frac{F}{K} - 1\right]\}$ as defined in (3). Then from Theorem 2, we have

$$
\begin{aligned}
S &\geq \left\lceil \frac{1}{L} \cdot \max\left\{ \sum_{i=1}^{K} \Big| \bigcap_{h=1}^{i} \mathcal{A}_{k_h} \Big| \;\Big|\; (k_1, \ldots, k_K) \in \mathcal{I} \right\} \right\rceil \\
&\geq \left\lceil \frac{1}{L} \cdot \left(|\mathcal{A}_1| + |\mathcal{A}_1 \bigcap \mathcal{A}_2| + \ldots + \Big| \bigcap_{k=1}^{K} \mathcal{A}_k \Big| \right) \right\rceil \\
&= \left\lceil \frac{1}{L} \cdot \frac{F}{K} \cdot ((K-t) + (K-t-1) + \cdots + 1) \right\rceil \\
&= \left\lceil \frac{F(K-t+1)(K-t)}{2LK} \right\rceil .
\end{aligned}
$$

Then the proof is completed. □

B. Achievable scheme for $L < \frac{KM}{N}$

For the case where $L \geq t := \frac{KM}{N}$, the authors in [6] proposed an MAPDA with the subpacketization $(L+t)K$ and sum-DoF $L+t$, which is the maximum sum-DoF under the MAPDA construction. Furthermore when $L + t = K$, the authors in [12], [13] proposed an MAPDA with supacketization K and the sum-DoF $L + t$. In this paper, we design two MAPDAs with linear subpacketization (but with a sum-DoF lower than the maximum one) for the case where $L < t$ and $t + L < K$.

First, by extending the MAPDA in [6] to the case $L < t$, we obtain the following MAPDA which achieves the sum-DoF $2L$ with subpacketization $2LK$.

Theorem 3. *For any positive integers K and t where $t \in [L+1 : K-L-1]$, there exists a $2L$-$(L, K, 2LK, 2Lt, K(K-t))$ MAPDA with the sum-DoF $2L$ and subpacketization $2LK$.*

Proof. We first focus on a cyclic star placement array $\mathbf{P}' = (p'_{f,k})_{f,k\in[K]}$ with dimension $K \times K$. Each entry $p'_{f,k} = *$ only if $k \in \{< f + z >_K | z \in [0 : t-1]\}$; thus row f contains Z neighbouring stars, which are in columns $< f + 0 >_K, < f + 1 >_K, \ldots, < f + t - 1 >_K$.

Next, we fill each null entry of $\mathbf{P}'$ by $2L$ different integers, through K rounds. Each round $r \in [K]$ contains $K - t$ sub-rounds. For each sub-round $j \in [K - t]$, we define two $2L$-vectors as

$$
\begin{aligned}
\mathbf{f}_j^r &=< [< L + j - [L] >_{K-t} + [L], \mathbf{e}_L] + r - 1 >_K, \quad (4)\\
\mathbf{k}_j^r &=< [[L], < [L] + j - 1 >_{K-t} + t] + r - 1 >_K, \quad (5)
\end{aligned}
$$

where $\mathbf{e}_L$ represents all 1 vectors with dimension $1 \times L$. For each $\ell \in [2L]$, denote the ℓ^{th} elements of $\mathbf{f}_j^r$ and $\mathbf{k}_j^r$ by $\mathbf{f}_j^r(\ell)$ and $\mathbf{k}_j^r(\ell)$, respectively. Then we append integer $2L(K-t)(r-1)+\ell$ into the entry in row $\mathbf{f}_j^r(\ell)$ and column $\mathbf{k}_j^r(\ell)$ of $\mathbf{P}'$. Since $t > L$, the elements in $\mathbf{k}_j^r$ are different; thus an integer cannot appear twice in one column, coinciding with the condition C3 in Definition 1.

Note that in each sub-round, we introduce one distinct integer into $2L$ entries of $\mathbf{P}'$, and that there are totally $K(K-t)$ sub-rounds. Hence, by the symmetry, there are $(K-t)2L$ integers in each column. Since there are t stars in each column, by the above construction, each non-star entry of $\mathbf{P}'$ contains $2L$ integers.

Finally, we can generate a $2L$-$(L, K, 2LK, 2Lt, K(K-t))$ MAPDA $\mathbf{P}$ with dimension $2L \times K$, from $\mathbf{P}'$. More precisely, we can replicate $\mathbf{P}'$ $2L$ times in the vertical direction, $\begin{pmatrix} \mathbf{P}' \\ \vdots \\ \mathbf{P}' \end{pmatrix}$. Then in the ℓ^{th} replicate where $\ell \in [2L]$, we replace each non-star entry (which is a vector of $2L$ integers) by the ℓ^{th} integer of this vector.

Due to the limitation of pages, we omit the decodabilty proof, which is similar to the one in [6].[2] □

Second, for the case $t \in [2L-1 : K-1]$, we propose the following scheme which can further reduce the subpacketiza-

[1] Otherwise, assume that $p_{f_1,k_1} = p_{f_2,k_2} = \cdots = p_{f_{L+1},k_{L+1}} = s$. Then p_{f_{L+1},k_1}, p_{f_{L+1},k_2}, …, $p_{f_{L+1},k_{L+1}}$ are all integers in $\mathbf{P}^{(s)}$, since

$$f_{L+1} \in \bigcap_{i=1}^{L+1} \mathcal{A}_{k_i} \subseteq \bigcap_{i=1}^{L} \mathcal{A}_{k_i} \subseteq \cdots \subseteq \bigcap_{i=1}^{2} \mathcal{A}_{k_i} \subseteq \mathcal{A}_{k_1}.$$

This contradicts Condition C4.

[2] Note that, in the construction of the scheme in [6] which is for the case $L \geq t$, different from (4) and (5), the definitions of $\mathbf{f}_j^r$ and $\mathbf{k}_j^r$ are $< [< t + j - [t] >_{K-t} + [t], \mathbf{e}_L] + r - 1 >_K$ and $< [[t], < [L] + j - 1 >_{K-t} + t] + r - 1 >_K$, respectively.

tion of the MAPDA in Theorem 3, where in Section IV we will present an example to illustrate its construction.

Theorem 4. *For any positive integers K and t where $t \in [2L-1 : K-L-1]$, there exists a $2L$-$(L, K, \alpha K, \alpha t, \frac{\alpha K(K-t)}{2L})$ MAPDA with the sum-DoF $2L$ and subpacketization αK where*

$$\alpha = \begin{cases} 2L & \text{if } 2 \nmid (K-t) \\ L & \text{if } 2|(K-t),\ L \nmid K \\ 1 & \text{if } 2|(K-t), L|K \end{cases}. \tag{6}$$

C. Performance analysis

By Theorem 3, we have a $2L$-$(L, K, \alpha K, \alpha t, S_{\text{Th}})$ MAPDA where $S_{\text{Th}} = \frac{\alpha K(K-t)}{2L}$ and $\alpha = 2L$. By Theorem 4 we have an MAPDA under the cyclic star placement with the same S_{Th}. In the following theorem, we show an order optimality result of the proposed MAPDAS in Theorems 3 and 4.

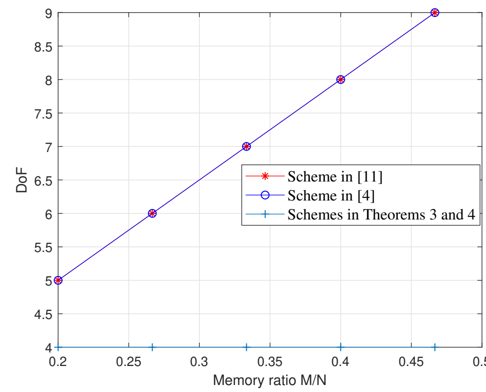
Theorem 5. *Given a (K, F, Z) cyclic star placement array where $\frac{Z}{F} \leq \frac{1}{2}$, the sum-DoF achieved by the proposed MAPDAs in Theorems 3 and 4 is order optimal within 2 under MAPDA construction for the case $M/N \leq 1/2$.*

Proof. By Corollary 1 if $F = \alpha K$ and $Z = \alpha t$ we have the lower bound $S \geq \lceil \frac{\alpha(K-t)(K-t+1)}{2L} \rceil$ for any MAPDA under the cyclic star placement. Then if $\frac{Z}{F} \leq \frac{1}{2}$ (i.e., $t \leq \frac{K}{2}$), we have

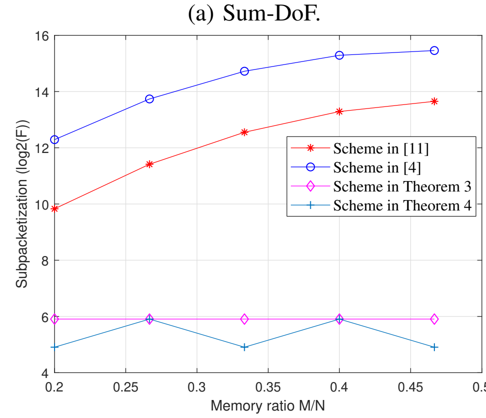
$$\frac{S_{\text{Th}}}{\lceil \frac{\alpha(K-t)(K-t+1)}{2L} \rceil} \leq \frac{\alpha K(K-t)/(2L)}{\alpha(K-t)(K-t+1)/(2L)} < 2.$$

Recall that the sum-DoF is equal to $\alpha K(K-Z)/S$. Hence, given a cyclic star placement with $F = \alpha K$, $t \leq \frac{K}{2}$ (i.e., $M/N \leq 1/2$), and $L < t = \frac{KM}{N}$, the sum-DoF of our schemes (i.e., $2L$) is at least half of the best sum-DoF under this cyclic star placement. □

Finally we compare the proposed schemes in Theorems 3 and 4 with the existing schemes in [4], [12]. In Fig. 1, we consider the case where $K = 15$ and $L = 2$. When $M/N = 0.2$, we can see that the schemes in [4], [12] have the maximum sum-DoF 5 with subpacketizations 5005 and 910, respectively. In this case, our scheme in Theorem 3 achieves the sum-DoF 4 with the subpacketization 60, while our scheme in Theorem 4 achieves the sum-DoF 4 with the subpacketization 30.



(a) Sum-DoF.



(b) Subpacketization.

Fig. 1: Cache-aided MISO broadcast problem with $K = 15, L = 2$.

IV. An Example for the MAPDA in Theorem 4

For any positive integers K and t where $K > t > 2$, we construct a $2L$-$(L, K, \alpha K, \alpha t, \frac{\alpha K(K-t)}{2L})$ MAPDA $\mathbf{P}$ under the cyclic star placement by the following two steps. First we construct a $K \times K$ base square cyclic star placement array $\mathbf{B}$ and obtain a star placement array $\mathbf{C}$ by vertically replicating $\mathbf{B}$ α times. Then we fill integers into the null entries of $\mathbf{C}$ to obtain the MAPDA $\mathbf{P}$. Due to the limitation of pages, we will take the case where $K = 7$, $t = 3$ and $L = 2$ as an example to illustrate our construction.

A. Construction of the star placement arrays $\mathbf{B}$ and $\mathbf{C}$

From (2) we can get a 7×7 base square cyclic star placement array $\mathbf{B} = (b_{f,k})_{f,k \in [7]}$ as follows,

$$\mathbf{B} = \left(\begin{array}{c|c|c|c|c|c|c} * & * & * & & & & \\ \hline & * & * & * & & & \\ \hline & & * & * & * & & \\ \hline & & & * & * & * & \\ \hline & & & & * & * & * \\ \hline * & & & & & * & * \\ \hline * & * & & & & & * \end{array}\right). \tag{7}$$

Recall that the position (f, k) of an entry in an array, represents that this entry is located at row f and column k of this array. The positions of all null entries in $\mathbf{B}$ are divided into the following disjoint orbits

$$\begin{aligned} \mathcal{B}_1: &= \{(<k+2>_7, <k+1>_7) \mid k \in [7]\} \\ &= \{(3,2),(4,3),(5,4),(6,5),(7,6),(1,7),(2,1)\}, \\ \mathcal{B}'_1: &= \{(<k>_7, <k+3>_7) \mid k \in [7]\} \\ &= \{(1,4),(2,5),(3,6),(4,7),(5,1),(6,2),(7,3)\}, \\ \mathcal{B}_2: &= \{(<k+3>_7, <k+1>_7) \mid k \in [7]\}, \\ &= \{(4,2),(5,3),(6,4),(7,5),(1,6),(2,7),(3,1)\}, \\ \mathcal{B}'_2: &= \{(<k>_7, <k+4>_7) \mid k \in [7]\} \\ &= \{(1,5),(2,6),(3,7),(4,1),(5,2),(6,3),(7,4)\}. \end{aligned}$$

In our following integer-filling strategy, we will put each integer into a pair of orbits, $\mathcal{B}_1$ and $\mathcal{B}'_1$, such that each integer appears exactly 4 times; and put each integer into a pair of orbits, $\mathcal{B}_2$ and $\mathcal{B}'_2$, such that each integer appears exactly 4 times too. Recall that each orbit has exactly $K = 7$ positions. Since to achieve the sum-DoF $2L = 4$, each integer needs to appear in the positions of $\mathcal{B}_1$ and $\mathcal{B}'_1$ exactly 4 times; thus we need to have $4|2 \times 7$, which is impossible. So we need to replicate the base square $\mathbf{B}$ twice which coincides with α in (6). Thus we can get the star placement array $\mathbf{C}$ of the objective MAPDA $\mathbf{P}$ by vertically replicating $\mathbf{B}$ $\alpha = 2$ times.

Clearly the positions of all the null entries in $\mathbf{C}$ can be represented by the orbits $\mathcal{B}_1$, $\mathcal{B}'_1$, $\mathcal{B}_2$ and $\mathcal{B}'_2$ as follows,

$$\begin{aligned}
\mathcal{C}_1 =&\{(< k+2 >_{14}, < k+1 >_7) \mid k \in [14]\}\\
=&\{(3,2),(4,3),(5,4),(6,5),(7,6),(8,7),(9,1),(10,2),\\
&(11,3),(12,4),(13,5),(14,6),(1,7),(2,1)\}, \qquad (8)\\
\mathcal{C}'_1 =&\{(< k >_{14}, < k+3 >_7) \mid k \in [14]\}\\
=&\{(1,4),(2,5),(3,6),(4,7),(5,1),(6,2),(7,3),(8,4),\\
&(9,5),(10,6),(11,7),(12,1),(13,2),(14,3)\}, \qquad (9)\\
\mathcal{C}_2 =&\{(< k+3 >_{14}, < k+1 >_7) \mid k \in [14]\}\\
=&\{(4,2),(5,3),(6,4),(7,5),(8,6),(9,7),(10,1),(11,2),\\
&(12,3),(13,4),(14,5),(1,6),(2,7),(3,1)\}, \qquad (10)\\
\mathcal{C}'_2 =&\{(< k >_{14}, < k+4 >_7) \mid k \in [14]\}\\
=&\{(1,5),(2,6),(3,7),(4,1),(5,2),(6,3),(7,4),(8,5),\\
&(9,6),(10,7),(11,1),(12,2),(13,3),(14,4)\}. \qquad (11)
\end{aligned}$$

B. *Construction of the MAPDA* $\mathbf{P}$

In the following, we will use an integer set to fill the null entries in $\mathcal{C}_1$, $\mathcal{C}'_1$, and use another integer set to fill the null entries in $\mathcal{C}_2$, $\mathcal{C}'_2$, such that we can construct $\mathbf{P}$ from $\mathbf{C}$.

Integer-filling: For each $j \in [2]$ and for each $k \in [\alpha K] = [14]$, we put integer $\lfloor \frac{k}{2} \rfloor + 7 \times (j-1) + 1$ into the positions $(< j+k+1 >_{14}, < k+1 >_7))$ of $\mathcal{C}_j$ and $(< k >_{14}, < j+k+2 >_7)$ of $\mathcal{C}'_j$, respectively.

Note that for each integer $k \in [14]$, the pairs (j,k) and $(j,k+1)$ lead to the same integer $\lfloor \frac{k}{2} \rfloor + 7 \times (f-1) + 1$. Hence, each integer appears $2L$ times in $\mathbf{P}$.

For instance when $k = 1$ we put the integer $\lfloor \frac{k}{2} \rfloor + 7 \times 0 + 1 = \lfloor \frac{1}{2} \rfloor + 1 = 1$ into the positions $(< 1+1+1 >_{14}, < 1+1 >_7) = (3,2)$ of $\mathcal{C}_1$ in (8) and $(< 1 >_{14}, < 1+1+2 >_7) = (1,4)$ of $\mathcal{C}'_1$ in (9), respectively. When $j = 2$ and $k = 11$, we put integer $\lfloor \frac{11}{2} \rfloor + 7 \times 1 + 1 = 13$ into the positions $(< 2+11+1 >_{14}, < 11+1 >_7) = (14,5)$ of $\mathcal{C}_2$ in (10) and $(< 11 >_{14}, < 2+11+2 >_7) = (11,1)$ of $\mathcal{C}'_2$ in (11), respectively.

Finally we can obtain the following 4-$(2,7,14,6,14)$ MAPDA,

$$\mathbf{P} = \left(\begin{array}{c|c|c|c|c|c|c}
* & * & * & 1 & 8 & 13 & 7\\
7 & * & * & * & 1 & 8 & 14\\
14 & 1 & * & * & * & 2 & 9\\
9 & 8 & 1 & * & * & * & 2\\
3 & 10 & 8 & 2 & * & * & *\\
* & 3 & 10 & 9 & 2 & * & *\\
* & * & 4 & 11 & 9 & 3 & *\\
\hline
* & * & * & 4 & 11 & 10 & 3\\
4 & * & * & * & 5 & 12 & 10\\
11 & 4 & * & * & * & 5 & 12\\
13 & 11 & 5 & * & * & * & 6\\
6 & 13 & 12 & 5 & * & * & *\\
* & 7 & 14 & 12 & 6 & * & *\\
* & * & 7 & 14 & 13 & 6 & *
\end{array}\right),$$

which leads to a coded caching scheme for the $(2,7,M,N)$ cache-aided MISO broadcast problem with memory ratio $\frac{M}{N} = \frac{3}{7}$, sum-DoF 4, and subpacketization 14, while the schemes in [2], [4], [12] achieve the sum-DoF 5 with subpacketizations 630, 105 and 175, respectively.

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